

D. Fermat and Huygens' constructions.

1. A lens, corrected for spherical aberration, is used to image a distant axial point source. The lens has diameter 100mm and focal length 500mm. Use Fermat's principle to decide how close to the focal point, on the axis, it will be possible to detect that the image is out of focus (This is called "depth of focus").

2. A GRIN (Gradient index) lens is made from a thin plate (thickness d in z -direction) of a material with refractive index varying in the (x, y) plane as:

$$n(x, y) = n_0 - \alpha(x^2 + y^2)$$

Use Fermat's principle to find the paraxial focal length of this lens.

3. Use Huygens' construction to understand self-focusing in a non-linear medium. In such a medium, the refractive index usually increases with the light intensity. Show by drawing the wave fronts that a local maximum in the light intensity concentrates the light onto itself, so that a "thread" of light is formed. What would happen if the refractive index *decreased* with the light intensity?

4. The refractive index of air as a function of temperature T ($^{\circ}\text{C}$) is given by $n(T) = 1.000191 - 1.0 \times 10^{-6}T$. Above a heated road, the air temperature is 80°C , although the ambient air temperature is 30°C . At what distance from an observer of height 1.80m does the road appear to be wet ("fata morgana")?